CALCULATION OF THE "INITIAL" THICKNESS OF THE "MICROLAYER" DURING BUBBLE BOILING

G. F. Smirnov

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The relationship between the initial thickness δ_0 of the microlayer and the local hydrodynamic characteristics is established on the basis of the proposed idealized model of the formation of the microlayer.

In reports [1-9 and others] it is shown that during the growth of a vapor bubble a thin layer of liquid (the "microlayer") forms at its base.

The transfer of heat through the "microlayer" in a number of cases makes an important contribution to the mechanism of heat exchange during bubble boiling.

The intensity of heat transfer through the "microlayer" depends essentially on its "initial"thickness δ_0.

A calculating equation for the "initial" thickness of the "microlayer" during boiling in a free volume is proposed in [8] and [3]:

$$\delta_0 = C_1 \sqrt{\nu' \tau}. \tag{1}$$

The constant is chosen from an analysis of experimental data. In [3] C_1 = 0.8 and in [8] C_1 = 1.27. The following empirical equation for boiling in a horizontal slot is proposed in [5]:

$$\frac{d\delta_0}{dR} = C_2 \left(\frac{\mu' U_0}{\sigma} \right)^{2/3} \frac{s}{R} . \tag{2}$$

In [10] it is proposed to calculate the thickness of a film during boiling in capillaries on the basis of a dependence of the type

$$\delta_0 = C_3 R \left(\frac{\mu' U_0}{\sigma} \right)^{1/2} . \tag{3}$$

In the general case the formation of a "microlayer" at the base of a vapor bubble can be considered as the result of the interaction of two zones of liquid: Zone I which moves together with the phase interface and Zone II of boundary layers of liquid; i.e., the "microlayer."

A similar hypothesis is advanced in [3] for boiling in a free volume.

In [5] such a condition of division of the moving layers of liquid into two zones during the formation and growth of the vapor cavity is confirmed experimentally, and it is established that the thickness δ_0 of the liquid "microlayer" formed does not vary with time if a process of evaporation is absent.

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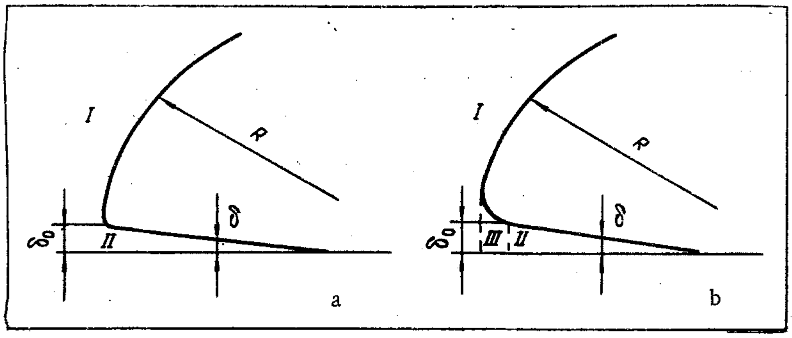

Fig. 1. Actual and idealized diagrams of the formation of the "initial" thickness of the "microlayer": a) ideal diagram; b) actual diagram; I, II) zones of "microlayer," III) transition zone.

Thus, the "initial" thickness δ_0 of the "microlayer" can be determined if: a) one takes the velocity at the boundary of the two zones as equal to the velocity of movement of the phase interface; b) one sets the pressure gradients in zones I and II as identical at fixed times.

A conditionally proposed model of the joint movement of the two zones of liquid and the actual model are presented in Fig. 1.

Actually there exists a transition zone III in which surface effects are manifested. A special analysis should show how much the δ_0 found in accordance with the idealized diagram of the motion differs from the true value of δ_0.

For the calculation of δ_0 we can use the equations of motion and of continuity. We will assume that the motion of the liquid is one-dimensional (radial).

Equation of motion:

$$\frac{\partial v_r}{\partial \tau} + v_r \frac{\partial v_r}{\partial r} = -\frac{1}{\rho'} \cdot \frac{\partial p}{\partial r} + \nu' \left(\nabla^2 v_r - \frac{v_r}{r^2} \right); \qquad (4)$$

equation of continuity:

$$\frac{\partial}{\partial r} (r v_r \delta_0) = 0; \qquad (5)$$

boundary conditions:

$$\begin{gathered} r = R;\ z = \delta_0;\ v_r = \dot{R} = \frac{dR}{d\tau}; \\ r = R;\ z = 0;\ v_r = 0; \\ 0 \leqslant r \leqslant R; \quad 0 \leqslant z \leqslant \delta_0. \end{gathered} \qquad (6)$$

We will assume that the velocity profile in the "microlayer" can be represented in the form

$$v_r = \dot{R} f_1(z) f_2(\tau). \qquad (7)$$

Let us adopt the simplest form of the function f_1 which satisfies the conditions (6):

$$f_1 = \frac{z}{\delta_0}. \qquad (7a)$$

Initial conditions: $f_2(0) = 1$ and $r = R$ at $\tau = 0$.

Substituting (7) and (7a) into (4), integrating over the thickness of the "microlayer" from 0 to δ_0, using (5) for the determination of $\partial v_r/\partial r$ and $\partial^2 v_r/\partial r^2$, and substituting into (4), at $\tau = 0$ we obtain

$$\ddot{R} - 2\dot{R}^2\left[\frac{1}{R} + \frac{1}{\delta_0}\cdot\frac{d\delta_0}{dR}\right] + \dot{R}\left[\frac{1}{\delta_0}\cdot\frac{d\delta_0}{d\tau} + \frac{df_2}{d\tau}\right] = -\frac{2}{\rho'}\cdot\frac{dp}{dR} + \nu'\dot{R}\left[\frac{1}{R\delta_0}\cdot\frac{d\delta_0}{dR} + \frac{2}{\delta_0^2}\left(\frac{d\delta_0}{dR}\right)^2 - \frac{1}{\delta_0}\cdot\frac{d^2\delta_0}{dR^2} - \frac{2}{\delta_0^2}\right]. \quad (8)$$

It is established from experiment [5] that in the absence of evaporation the "initial" thickness δ_0 of the "microlayer" does not vary with time and also that $\delta_0 \ll R$, and therefore, retaining in (8) values of the same order, we obtain

$$\ddot{R} - \frac{2}{3}\dot{R}^2\left[\frac{1}{R} + \frac{1}{\delta_0}\cdot\frac{d\delta_0}{dR}\right] = \frac{-1}{\rho'}\cdot\frac{dp}{dR}2 - \frac{2\nu'\dot{R}}{\delta_0^2}. \quad (9)$$

For the case of $1/\delta_0 \cdot d\delta_0/dR \ll 1/R$ we obtain a calculating equation for determining the "initial" thickness δ_0 of the "microlayer" for a known law of growth of the vapor cavity:

$$\delta_0 = \sqrt{\frac{2\nu'\dot{R}}{-2\frac{1}{\rho'}\cdot\frac{dp}{dR} - \ddot{R} + \frac{2}{3}\cdot\frac{\dot{R}^2}{R}}}. \quad (10)$$

The value of dp/dR is determined from the condition of motion of the phase interface.

With growth of the vapor bubble in a free volume we find, using the Rayleigh equation, that

$$\frac{p - p_\infty}{\rho'} = R\ddot{R} + \frac{3}{2}\dot{R}^2 + \frac{2\sigma}{\rho' R}; \quad (11)$$

when p_∞ = const we have

$$\frac{1}{\rho'}\cdot\frac{dp}{dR} = 4\ddot{R} + \frac{R\dddot{R}}{\dot{R}} - \frac{2\sigma}{\rho' R^2}. \quad (12)$$

Thus, for a bubble growing in a free volume

$$\delta_0 = \sqrt{\frac{2\nu'\dot{R}}{-9\ddot{R} - \frac{2R\dddot{R}}{\dot{R}} + \frac{4\sigma}{\rho' R^2} + \frac{2}{3}\cdot\frac{\dot{R}^2}{R}}}. \quad (13)$$

If for the stage of "rapid growth" of the bubble one neglects the surface forces and takes the law of growth in the form $R = C_1\tau^n$ in accordance with [3] then

$$\delta_0 = \sqrt{\frac{2\nu'\tau}{9(1-n) + 2\left(\frac{1}{n} - 1\right)(n-2) + 0.66n}}. \quad (14)$$

It follows from Eq. (14) that the actual value of δ_0 corresponds to $0.36 < n < 1.1$, which is in agreement with experiment. It also follows from (14) that the law

$$\delta_0 = C_0 \sqrt{\nu' \tau} \tag{15}$$

is sufficiently universal and valid for any real value of n, and not only for n = 0.5 as obtained in [3].

When n = 0.5 we have $C_0 = 1.04$, which is sufficiently close to the experimental value of 0.8 [3]. If the condition $1/\delta_0 \cdot d\delta_0/dR \ll 1/R$ is not observed then $C_0 < 1.0$.

In the case of "slow" growth of the bubble, when the inertial forces can be neglected compared with the surface forces, we obtain from Eq. (13)

$$\sigma_0 = 0.7R \sqrt{\frac{\mu' \dot{R}}{\sigma}}. \tag{16}$$

Equation (16) agrees with Eq. (3) proposed in [10] with the condition that $U_0 = \dot{R}$, while R is the variable radius of the growing vapor cavity.

In order to verify the applicability of Eq. (10) to a calculation with boiling in a flat slot in accordance with the experimental data [12] it is necessary to obtain an equation of the type of (12) for this case.

A strict solution of this problem goes beyond the bounds of the present work. For an approximate analysis we assume that the symmetrical growth of a cylindrical cavity is realized in a flat horizontal slot with radius B.

Taking the nonsteady motion of the liquid in the slot as laminar, assuming the velocity distribution obeys the law

$$v_r = \frac{3}{2} \cdot \frac{R\dot{R}}{r}\left[1 - \left(\frac{z}{s}\right)^2\right], \tag{17}$$

using the one-dimensional Navier–Stokes equation, and integrating over the thickness of the moving layer of liquid from z = −s to z = +s and over the radius r from the moving phase interface r = R to the exit from the slot r ≐ B, we obtain an approximate equation for the difference between the pressure p_R in the vapor cavity and the pressure p_B in the liquid at the exit from the slot:

$$p_R - p_B = \rho' \left\{(R\ddot{R} + \dot{R}^2)\ln\frac{B}{R} - \frac{2}{3}(\dot{R}R)^2\left[\frac{1}{R^2} - \frac{1}{B^2}\right] + \frac{3\nu'\dot{R}R}{s^2}\ln\frac{B}{R}\right\}. \tag{18}$$

With allowance for the surface effects and the pressure drop at the exit from the slot we have

$$-\frac{1}{\rho'} \cdot \frac{dp}{dR} = -\left[3\ln\frac{B}{R} + 0{,}2\left(\frac{R}{B}\right)^2 - 2{,}2\right]\ddot{R} - \frac{R\dddot{R}}{\dot{R}}\ln\frac{B}{R} + \\ + \frac{\dot{R}^2}{R}\left[1 - 0{,}2\left(\frac{R}{B}\right)^2\right] - \frac{3\nu'}{s^2}\dot{R}\left[\ln\frac{B}{R} - 1 + \frac{\ddot{R}R}{\dot{R}^2}\right] + \frac{\sigma}{\rho' R^2}. \tag{19}$$

The joint solution of Eqs. (10) and (19) with a known law R = f(τ) gives the value of the thickness of the microlayer.

In the general case, Eqs. (10) and (19) must be supplemented with the equation of state of the growing vapor cavity and the heat exchange equation.

TABLE 1. Comparison of Results of Calculation of δ_0 by Eqs. (10) and (19) with Data of [12]

R in mm	2,0	3,0	4,0	5,0
δ_0 according to [12] in μ	7,1	8,7	10	11,2
δ_0 from Eqs. (10) and (19), in μ	16,0	17,0	17,2	17,5

In the particular case when the law $R = f(\tau)$ or $\dot{R} = \varphi(\tau)$ is known, having determined $\ddot{R}$ and $\dddot{R}$ by graphic differentiation one can, using (10) and (19) directly, calculate the law of variation of δ_0 and compare it with experiment.

Experimental data which allow one to perform such a calculation are presented in [12].

An example of the calculation of the thickness of the evaporating microlayer during boiling in a flat horizontal slot and a graph of the dependence of $\dot{R}$ on time and on R are presented in [12].

We determined $\ddot{R}$ and $\dddot{R}$ by graphic differentiation and calculated the values of δ_0 from (10) and (19). In doing this it was kept in mind that calculation using (19) is admissible when $R \gg 2s$.

The results of the calculation by (10) and (19) and the data of [12] are presented in Table 1. The excess of the results of the calculation by Eqs. (10) and (19) and the data of [12] are presented in Table 1. The excess of the results of the calculation by Eqs. (10) and (19) compared with the data of [12] is connected not only with the error of the graphic differentiation and with the approximate nature of Eq. (19) but also with the inaccuracy of the values of δ_0 according to [12]. The values of δ_0 in [12] are obtained by a calculation based on an experimental law of growth of the "dry" spot during the evaporation of the microlayer. In the experiments of [12] this law is obtained for sizes of the "dry" spot with radius $0 \leq R_d \leq 0.4$ mm and cannot be reliably extrapolated (as is done in [12]) to the region of $0.5 \text{ mm} < R < 5 \text{ mm}$.

It must be noted that in the derivation of Eq. (10) it was assumed that the formation of the microlayer takes place on a smooth surface. Since the thickness δ_0 in many cases is a value on the order of 1-10 μ, which is quite commensurate with the absolute values of the roughness even for rather "smooth" surfaces, Eq. (10) requires correction for heat-exchange surfaces having an absolute roughness of more than 1 μ.

Thus, it follows from the material presented above that the idealized model proposed for the calculation of the initial thickness of the "microlayer" and leading to Eq. (10) can be used for an approximate quantitative analysis of the mechanism of the boiling process on "smooth" surfaces under those conditions when heat transfer through the microlayer makes an important contribution to the process.

NOTATION

δ_0, "initial" thickness of "microlayer," m; ν', coefficient of kinematic viscosity of liquid, m^2/sec; τ, time, sec; R, radius of phase interface, m; μ', coefficient of dynamic viscosity of liquid, kg/m·sec; $U_0 = \dot{R} = dR/d\tau$, velocity of motion of phase interface, m/sec; σ, surface tension coefficient, N/m; 2s, width of horizontal slot, m; v_r, current value of radial velocity, m/sec; ρ', density of liquid, kg/m^3; $\ddot{R} = d^2R/d\tau^2$, acceleration of liquid at phase interface, m/sec^2; $\dddot{R} = d^3R/d\tau^3$; p, pressure in liquid at phase interface, N/m^2; p_∞, pressure in free volume of liquid, N/m^2; B, external radius of horizontal slot, m; R_d, radius of "dry" spot, m.

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